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MEASUREMENT OF REPULSIVE QUANTUM VACUUM FORCES

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ABSTRACT

Quantum electrodynamics predicts that empty space (the quantum vacuum) contains a large amount of energy that corresponds to the lowest energy state (energy >0) of the electromagnetic field. Surfaces in the vacuum can experience forces that arise from the disturbance in the vacuum energy. The presence of attractive "Casimir" forces between uncharged, parallel, metal plates has been accurately verified in the last several years. Theoretical calculations have suggested the presence of repulsive vacuum forces for certain geometrical configurations. Here we describe an experiment in progress that is designed to determine if repulsive vacuum forces exist. In the experiment we measure the force exerted on a 200 um diameter metallized sphere mounted on an Atomic Force Microscope (AFM) that is placed very close to an array of gold microcavities. Observing a repulsive force on the sphere would verify the existence of repulsive forces. The ability to create attractive and repulsive vacuum forces by means of the geometry of the surfaces may permit the construction of devices that use ubiquitous vacuum energy in ways that assist with the space travel mission of NASA.

INTRODUCTION

Understanding the nature of vacuum forces and vacuum energy and how to manipulate this energy to obtain desired forces is a prerequisite to using these ubiquitous natural resources in any space application[1]. The theory for vacuum forces and quantum vacuum energy comes from Quantum Electrodynamics (QED), the theory of the interaction of matter and light[2]. **The role of the quantum vacuum is pervasive in modern physics. For example, it is involved in the calculation of atomic energy levels, the magnetic moment of the electron, the mass of elementary particles, spontaneous emission, dispersion forces between molecules, the large-scale structure of space-time.**

The experiment discussed in this paper is part of a three-year effort to begin to build, step by step, the knowledge base necessary for vacuum engineering. Our objective is to develop theoretical models of elementary systems that utilized vacuum forces and energy, to understand how these models behave, and then to explore some of these models experimentally. Since the critical dimensions required for these devices are typically micron to submicron, the experimental research utilizes microfabrication technology, and the methods developed for MicroElectro-Mechanical Systems (MEMS).

In space applications the application of vacuum energy systems might be power generation, propulsion itself, or the manipulation of the metric of space-time itself by the creation of regions of positive and negative energy density[3]. It is too early to determine if such developments are possible or to be able to clearly determine the role of vacuum energy in future space applications. If we can develop technologies for space travel that utilize vacuum energy, it is very convenient since this energy is pervasive throughout the universe.

Fifty years ago, Casimir predicted that the modifications to the vacuum energy arising from the presence of two uncharged, parallel, metal plates would cause the plates to attract each other. This attractive Casimir force varies as the inverse fourth power of the separation. At a separation of 10 nm the force/area is about 1 atm. In 1997 the prediction of Casimir was verified for the first time. In 1998 precision measurements corroborated the predictions of an attractive vacuum force between neutral parallel plates to an accuracy of several percent. **In March 2001, scientists at Lucent Technology used attractive parallel plate vacuum forces (Casimir forces) to actuate a MEMS torsion device[4]. Other MEMS devices using vacuum energy have been proposed.** [5]

Recent calculations have indicated that forces due to the quantum vacuum predicted by QED depend very strongly on the geometry of surfaces. For certain rectangular metal cavities, QED predicts the existence of repulsive forces on the walls of the cavity[6]. **In this paper we describe the current status of the first experiment specifically designed to measure repulsive forces due to modifications in vacuum energy density achieved by using metal surfaces.** The vacuum force is measured by means of an Atomic Force Microscope using a 200 um diameter metallized ball placed on the end of a calibrated cantilever. Our model suggests that a repulsive force on the ball would be observed when it approaches within 10's of nanometers from the top of an array of rectangular cavities, each of which is 100nm across and 1 um deep, patterned in gold using x-ray photolithography (Figure 2). For small separations between the surface of the sphere and the top of the cavity array, we are approximating an array of closed cavities, which, according to a QED calculation, exhibit repulsive forces. **The force between the sphere and the cavity array is modeled numerically, with heuristic approximations, to be compared to the measured force. It is important to note that no rigorous method has yet been developed to calculate the vacuum force between any two non-planar conducting surfaces using QED.** Only the parallel plate problem has been solved. Indeed there is some disagreement that a repulsive Casimir force should ever be present between two separate bodies[7]. Measurement of repulsive forces between separate conducting bodies may be expected to stimulate new developments in QED.

If our experiment verifies the existence of repulsive vacuum forces, then it may be possible to utilize repulsive forces as well as attractive vacuum forces in microelectro-mechanical systems (MEMS). The existence of attractive and repulsive Casimir forces might permit the development of a variety of novel MEMS devices of potential use to NASA.

Quantum Electrodynamics (QED), Vacuum Energy and Casimir Forces

Quantum Electrodynamics (QED), the theory of the interaction of electromagnetic fields and matter, has made predictions of atomic energy levels and electron magnetic moments that have been verified to 1 part in 10^{12}, which makes QED the most precisely verified theory in science[8,9]. In order to achieve this accuracy, QED predictions have to include the interaction of matter with "empty space" or, more accurately, the quantum vacuum[10].

Some predictions of QED are less enthusiastically received by the physics community than others. One of the confounding predictions of QED is an energy density in empty space that is many orders of magnitude greater that the energy density of matter itself[11]. For years this feature of QED was dismissed as of no physical significance. However, observable forces can result when surfaces are present that alter this vacuum energy density. About 50 years ago, Phillips Laboratory physicist H.G.B. Casimir predicted the presence of an attractive quantum vacuum force between neutral, parallel, metal plates[12]. In the last three years, experiments have accurately confirmed this prediction of QED for the first time, verifying the existence of attractive vacuum forces between conductive surfaces[13,14,4]. The parallel plate Casimir force goes as the inverse fourth power of the separation between the plates. At a separation of 100 nm the predicted force/area is equivalent to about 10^{-4} atm.; at 10 nm it is about 1 atm.

BACKGROUND

Since most aerospace researchers do not have backgrounds in quantum systems, we provide a brief background to motivate our study. It is certainly not obvious that there should be any energy at all in empty space, much less a very large amount! Nor is obvious why there should be forces due to the vacuum fluctuations. The evidence for this theoretical conclusion lies in numerous well verified experiments on atomic energy levels, the magnetic moment of the electron, the behavior of liquid helium, and the scattering of elementary particles[15].

Vacuum energy is a consequence of the quantum nature of the electromagnetic field, which is composed of photons. A photon of frequency ω has energy $\hbar\omega$, where $\hbar$ is Planck's constant. **The quantum vacuum can be interpreted as the lowest energy state (or ground state) of the electromagnetic (EM) field that occurs when all charges and currents have been removed, and the temperature has been reduced to absolute zero. In this state no ordinary photons are present. Nevertheless, because the electromagnetic field is a quantum system, like an atom, which has internal motion even at absolute zero, the energy of the ground state of the EM field is NOT zero.** Although the average value of the electric field $<E>$ vanishes in the ground state, the Root Mean Square

(RMS) of the field <E^2 > is **not** zero. Similarly the RMS of the ground state magnetic field <B^2> is not zero. Therefore the electromagnetic energy in the ground state, which from classical electrodynamics is proportional to <E^2> + <B^2>, is not zero. **A detailed theoretical calculation tells us that the electromagnetic energy in each mode of oscillation with frequency _ is ½○_, which equals one half of the amount of energy that would be present if a single "real" photon of that mode were present.** Adding up ½○_ for all possible modes of the electromagnetic field gives a very large number for the vacuum energy E_0 in the quantum vacuum:

$$E_o = \tfrac{1}{2}\sum_i \hbar\omega_i \tag{0.1}$$

The resulting vacuum energy E_0 is infinity unless a high frequency limit is used.

Inserting surfaces into the vacuum causes the modes of the EM field to change. This change in the modes that are present occurs since the electromagnetic field must meet the appropriate boundary conditions at each surface[16]. **Surfaces alter the modes of oscillation and therefore the surfaces alter the energy density corresponding to the lowest state of the EM field.** In actual practice, the modes with frequencies above the plasma frequency do not appear to be significantly affected by the metal surfaces since the metal becomes transparent to radiation above this frequency. In order to avoid dealing with infinite quantities, the usual approach is to compute the finite change in the energy of the vacuum ƛ_0 due to the presence of the surfaces[17]:

$$\Delta E_0\begin{bmatrix} change\cdot in\cdot vacuum\cdot \\ energy\cdot due\cdot to\cdot sufaces\end{bmatrix} = E_0\begin{bmatrix} energy\cdot in\cdot \\ empty\cdot space\end{bmatrix} - E_S\begin{bmatrix} energy\cdot in\cdot space\cdot \\ with\cdot surfaces\end{bmatrix} \tag{0.2}$$

where the definition of each term is given in brackets. This equation can be expressed as a sum over the corresponding modes:

$$\Delta E_0(due\cdot to\cdot sufaces) = \tfrac{1}{2}\sum_n^{\substack{empty\\space}} \hbar\omega_n - \tfrac{1}{2}\sum_i^{\substack{surfaces\\present}} \hbar\omega'_i \tag{0.3}$$

The quantity ΔE_0, which is the change in the vacuum energy due to the presence of the surfaces, can be computed for various geometries. The forces F due to the quantum vacuum are obtained by computing the change in the vacuum energy for a small change in the geometry. For example, consider a hollow conducting rectangular cavity with sides a_1, a_2, a_3. Let en(a_1, a_2, a_3) be the change in the vacuum energy due to the cavity, then the force F_1 on the side perpendicular to a_1 is:

$$F_1 = -\frac{\partial en}{\partial a_1} \tag{0.4}$$

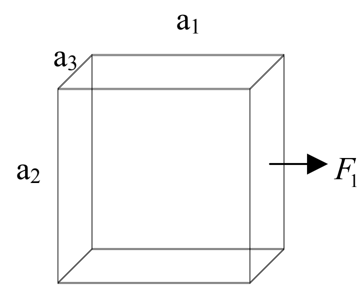


Figure 1. Geometry of rectangular cavity.

Equation (0.4) also represents the conservation of energy when the wall perpendicular to a_1 is moved infinitesimally[18]:

$$\delta en = -F_1\delta a_1 \tag{0.5}$$

Thus if we can calculate the vacuum energy as a function of the dimensions of the cavity we can compute derivatives which give the forces on the surfaces. For uncharged parallel plates with a large area A, very close to each other, this equation predicts an attractive or negative force between the plates:

$$F_{att} = -\frac{\pi^2}{240}\frac{\hbar c}{d^4}A \tag{0.6}$$

This force is called the parallel plate Casimir force, which was measured in three different experiments in the last three years[13,14,4]. The

Casimir force has only been computed and measured for this very large parallel plate geometry.

QED makes some unexpected predictions about Casimir forces in other geometries that have never been verified. For conductive rectangular cavities, the vacuum forces on a given face can be repulsive (positive), attractive (negative), or zero depending on the ratio of the sides[6]. **We are particularly interested in measuring these repulsive Casimir forces. Verifying the existence of such forces would have important implications in quantum electrodynamics and would be an important step to utilizing Casimir forces in a variety of MEMS devices.**

DESCRIPTION OF THE EXPERIMENT

Atomic Force Microscope

It is not practical to directly measure the force on one wall of a submicron metallic cavity. Hence another approach is needed. We chose to use an Atomic Force Microscope (AFM), which can provide a very sensitive measure of forces into the piconewton range (10^{-12} Newton). The AFM employs a 300 um long micromachined silicon nitride cantilever with a 200 um metallized sphere on the end that can be used to probe the vacuum energy density in the neighborhood of a rectangular micromachined cavity with no top surface[19] (Figure 2). When the sphere experiences a force, the cantilever is deflected. The deflection of the sphere is measured by shining a laser diode onto the reflective surface of the cantilever. The reflected light is collected in a photodiode that is divided into two adjacent regions. As the spot of light moves during a deflection, the ratio of current from the two regions changes, giving a sensitive quantitative measure of the cantilever deflection. It is possible to measure deflections of several nanometers in this manner. The cantilever is calibrated by determining the cantilever deflection for a known electrostatic force. The high precision of this experiment is made possible by the use of a Molecular Imaging AFM system that was specially developed at the University of Alabama at Huntsville for vacuum operation. With the very small distance (much less than the mean free path of the molecules) between the sphere and cavity, gas molecules can become effectively trapped, taking hours to remove under vacuum. For the most reliable measurements it is necessary to remove the trapped molecules and operate at a sufficiently low vacuum. Trapped molecules may result in a squeeze film damping force because the cantilever is always vibrating slightly.

The AFM stage was connected to a vacuum flange with the necessary feedthroughs. The sample is mounted and aligned when the AFM is in air (Figure 3a). Then the AFM is inserted into the vacuum chamber and the flange bolted in place using copper gaskets (Figure 3b). The vacuum without the AFM is 10^{-8} torr; inserting the AFM reduces the vacuum to about 10^{-4} torr. The system is pumped with a turbo molecular pump and an ion pump. The AFM is housed in a clean room environment.

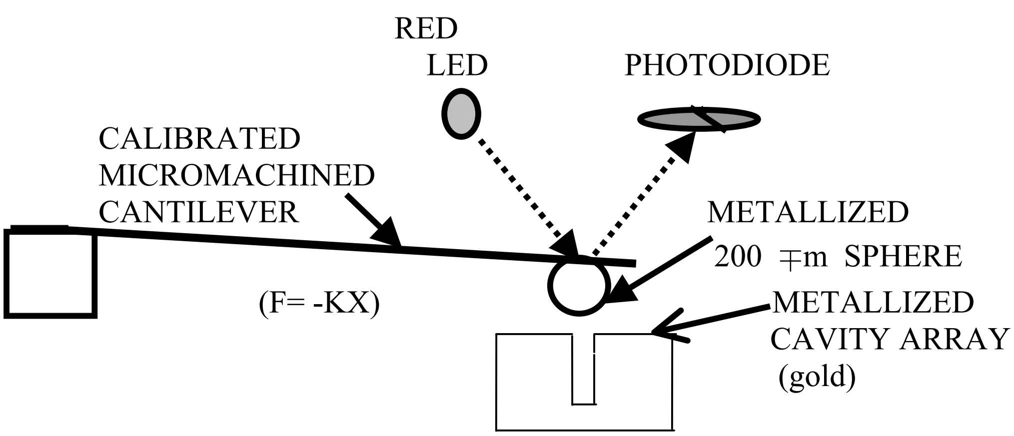
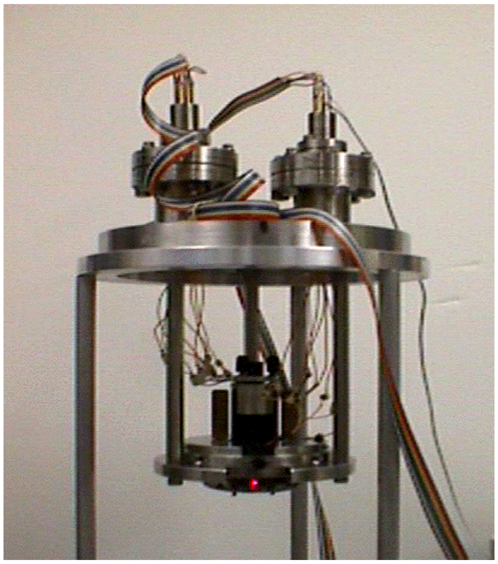


Figure 2. Schematic of Atomic Force Microscope measurement of the vacuum force between a metallized sphere on a cantilever and a rectangular cavity etched in gold (not to scale).

The force constant of the AFM cantilever is measured by using electrostatic forces. A know potential is applied between the test surface and the cantilever, and the deflection of the cantilever due to this potential is measured. The corresponding force is calculated using a finite element classical electrodynamic calculation. The system was tested by making measurements on the attractive Casimir force between the metallized sphere and a flat gold region and comparing these results to know values.



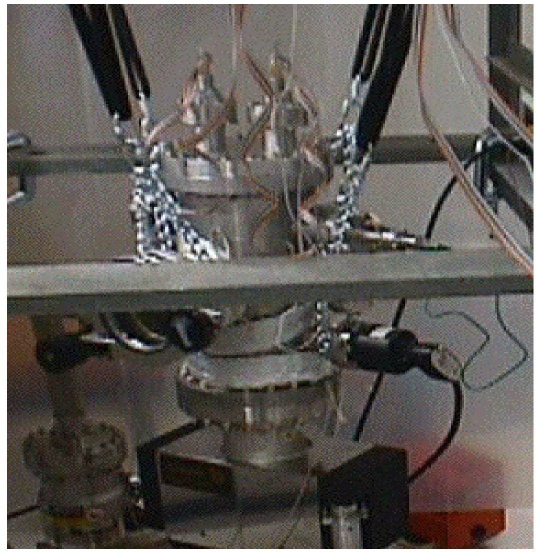


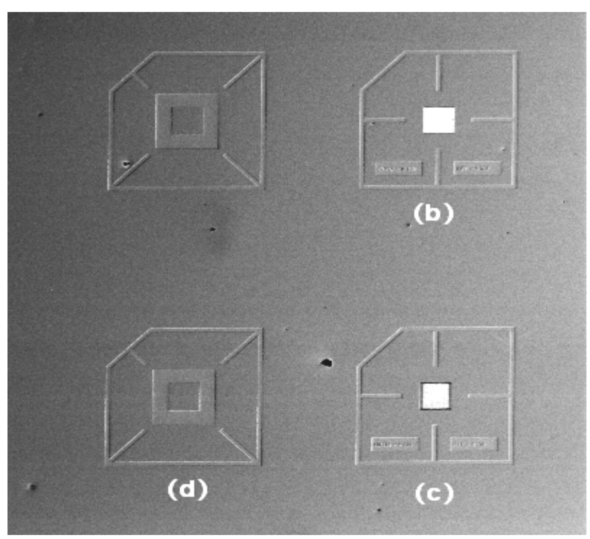
Figure 3. (a) shows the AFM stage below the vacuum flange, connected to the vacuum feedthroughs above. (b) shows the AFM mounted inside a small, stainless steel, vacuum chamber supported by elastic cords to reduce vibration.

Cavity Design

Numerical computations were done of the change in the vacuum energy and vacuum forces for a variety of rectangular cavities using QED methods[1,6]. The goal was to determine a cavity geometry that 1) would yield a large, detectable, repulsive force, 2) the repulsive force would change slowly with distance, and 3) that could be fabricated. The second requirement was deemed advisable to insure that the repulsive vacuum force would not vary too rapidly as the distance between the sphere and the opening of an etched rectangular cavity changed. The final cavity design selected was 0.1 um x 100 um x 1 um (width x length x depth), with walls that are 0.1 um thick.

Wisconsin Center for X-Ray Lithography. The cavity arrays fabricated are 100um x 100 um square, with cavity walls 0.5 um deep, with thickness t between 250 and 300 nm thick, and cavity widths w between 125 and 150 nm. Calibration surfaces for the AFM were also included in the design. The overall test pattern design is shown in Figure 4, and a portion of one of the cavity arrays closest to the target design is show in Figure 5.

Figure 4. A SEM photograph of a portion of the test die, showing two 500 um square calibration patterns on the left, and two 500 um square cavity array regions on the right side. The center of each calibration pattern is a flat gold surface at the same level as the bottom of the cavities, surrounded by a gold surface at the level of the top of the array. The white regions on the right are the 100 x 100 um cavity arrays. The rectangular regions below the arrays indicate the nominal cavity width and wall thickness (50 x).



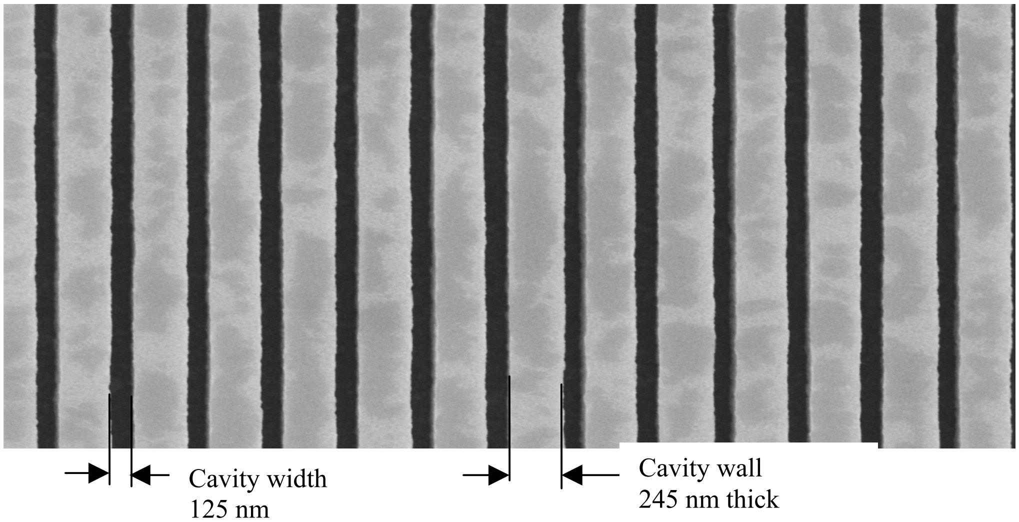


Figure 5. SEM photograph of portion of the gold cavity array. Each cavity is actually100 um long. The entire array is 100 um wide (magnification 37,000; 15kV).

Theoretical Modeling

As mentioned previously, no QED method has been developed to compute the vacuum forces between separate conducting surfaces. There is no theoretical model for such a configuration of two separate surfaces in the literature; no QED calculation of Casimir forces have been done except for planar or slightly rough planar surfaces. Hence we developed a heuristic model in which we assume the force on the sphere arises from two effects: 1) the attractive force due to the proximity of the sphere to the flat top surfaces of the cavity walls (parallel plate attractive Casimir force Eq 1.6), and 2) the repulsive forces on the sphere due to the cavity. The geometry of the experiment is shown in Figure 4.

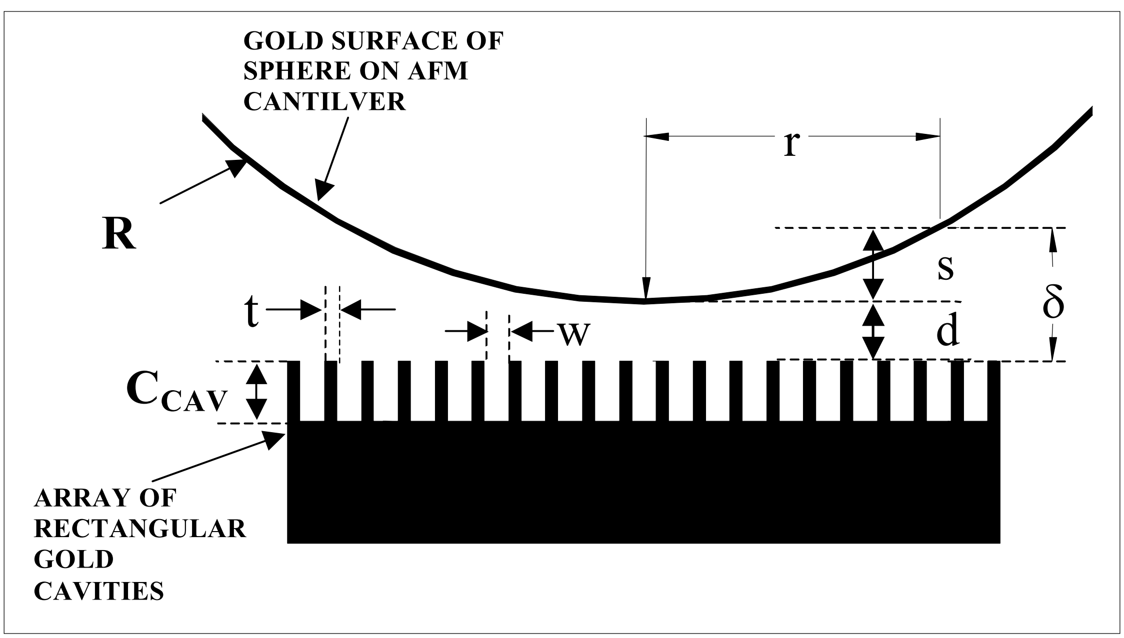


Figure 6. Distance definitions, illustrating actual cavity depth (C_{CAV}), individual cavity width (w), cavity wall thickness (t), separation distance (*d*), local sag (*s*), and local separation distance (δ).

The sphere has radius R. The closest point or tangent point of the sphere is located a distance d above the tops of the cavities. The local separation distance is $\delta = d + s$, where s is the local sag of the spherical surface. The quantity δ can be given as a function of r, the horizontal distance from the tangent point of the sphere. The expression for the attractive Casimir force Eq. 0.6 is actually derived for an infinite parallel plate geometry in which the lateral dimensions of the plates are much bigger than the spacing between the plates. Since this condition is not met for spacing d>t (width of cavity wall), we applied a correction factor to the attractive force, obtaining the expression for the attractive force:

$$F_{att}(r) = \frac{\pi^2}{240}\frac{\hbar c}{\delta(r)^4} A_w \left(\frac{t}{t+\delta(r)}\right) \qquad 0.7$$

where we used the local separation distance $\delta(r)$, □□ t is the thickness of the cavity wall, and A_w is the area of the tops of the cavity walls. The QED calculation of the repulsive Casimir force was for a closed, rectangular metallic box. Therefore we need a method to correct for the experimental geometry in which there is a gap at the top of the box. For the repulsive cavity force, we used the computed force for a closed cavity of width "w" with a depth equal to the actual depth (C_{CAV}), and multiplied it by an approximate correction factor K(r) suggested by theoretical analysis[20]:

$$\mathrm{K(r)} = \left(\frac{C_{CAV}}{C_{CAV}+\delta(r)}\right)^3 \left(\frac{w}{w+\delta(r)}\right) \qquad 0.8$$

Eqs. 0.7 and 0.8 predict that both forces decay rapidly as the separation distance is increased. Rather than sum over individual cavities, we used an effective pressure distribution, which is an area-weighted combination of the cavity force and wall force. This provides a pressure distribution $p(r)$ over the surface of the sphere, where r is the radial coordinate. This pressure distribution depends on the geometry, including the separation distance, d.

To obtain the total force on the sphere, we integrated the pressure $p(r)$ on the bottom of the sphere due to the sum of the forces, i.e.,

$$F = \int_0^{R/2} 2\pi r p(r) dr \qquad 0.9$$

Figure 7 shows a plot of the force F as a function of the separation d for a cavity array with 1) the target dimensions, namely w= 0.1 um wide with t=0.1 um thick sidewalls and 1.0 um deep, and 2) the best actual cavity dimensions fabricated, namely w=0.125 um wide cavities with t=0.250 um sidewalls, 0.5 um deep.

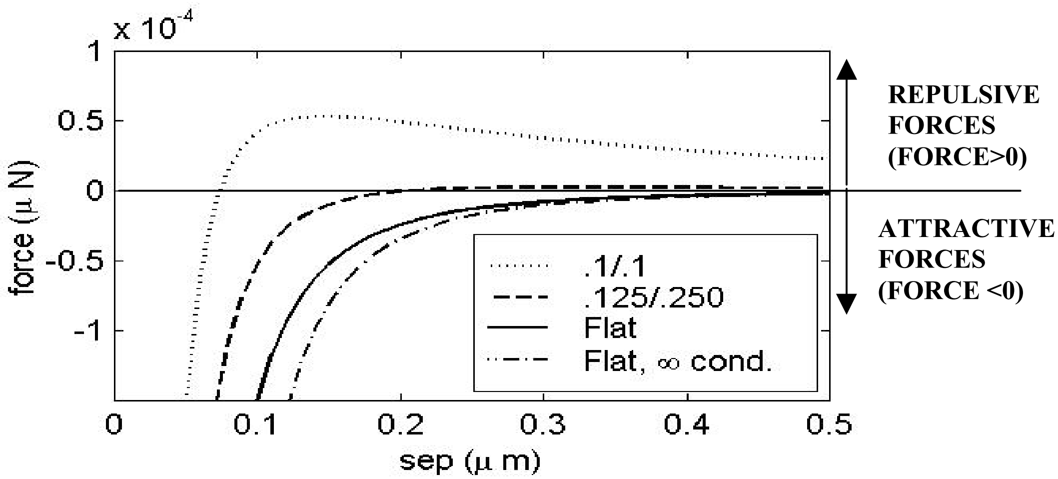


Figure 7. Force vs. distance d for a 100 um square cavity array with 1) the target dimensions, cavity width w = 0.1 um, wall thickness t=0.1 um, depth Ccav = 1.0 um deep; 2) the best fabricated cavity, width w=0.125 um, thickness 0.250, depth 0.5 um deep. The two uppermost curves show the force for the sphere above the cavity arrays. The solid curve shows the attractive force for a flat surface, with no cavities. Conductivity corrections are included. The sphere diameter is 210 um.

Also shown for comparison, is the calculated force (labeled Flat in the figure) for the case of a sphere over a flat surface, i.e. with no cavities at all in the gold. The known correction factors for finite conductivity for the force between parallel plates were used except for the curve labeled infinite conductivity. For the lack of any other theory, the same conductivity correction factors were applied to the cavity geometry.

For the parallel plate case, the force is always negative or attractive, and the force decreases rapidly (Eq 1.7 integrated over the hemisphere, or approximately as $-1/\delta^{4}$) with the separation δ. The component with upward curvature in the cavity array curve is due to the repulsive force (Eq 1.8 integrated over the hemisphere, which goes approximately as $1/\delta^{3}$). Because of the more rapid inverse variation of the attractive force, the attractive force dominates at very small separations d, going to zero more rapidly than the attractive force. Hence the repulsive force dominates at larger separations, above about 0.1 um. From QED we expect that smaller cavities would give larger repulsive forces; and thinner walls give smaller attractive forces. The predictions of the model show this desired behavior, and the model calculations appear to go smoothly into these known curves for flat surfaces. It appears that the repulsive force is on the edge of detectability for the cavities fabricated.

CONCLUSIONS

Quantum electrodynamics, which has made predictions which have been verified to 1 part in 10^{12}, predicts the existence of a large, ubiquitous, zero-point vacuum energy density in empty space. The question arises: Can we make use of this energy in some way to facilitate space travel, such as energy generation, propulsion, or creation of wormholes? It is too early to determine if such developments are possible or to be able to clearly determine the role of vacuum energy in future space applications. Our investigation begins with what we do know about vacuum energy, and extends those boundaries. We know that QED predicts that as a consequence of this energy, an attractive force will exist between uncharged, parallel, metal plates. In the last few years, accurate measurements have confirmed the existence of this force. QED also predicts the existence of repulsive forces in small rectangular metal boxes in which one dimension is much less than at least one of the other two dimensions. Although no one has done a rigorous calculation, it appears probable, based on theory, that a repulsive force should exist between two separate surfaces that closely approximate such a closed box. Vacuum forces that are repulsive because of the geometry have never been observed.

We have designed an experiment to measure repulsive vacuum forces. A model has been developed to predict the vacuum force on a metallized sphere attached to a cantilever on an AFM when the sphere is brought to within nanometer distances of an array of gold cavities. An AFM that operates in vacuum at 10^{-4} torr has been constructed to perform the experiment. Based on our model calculations, it appears we should be able to measure repulsive vacuum forces using the AFM, provided the cavities have small enough dimensions. Based on our model calculations (Figure 7) the cavities fabricated to date (125 nm width, 245 nm wall thickness) have dimensions that may be too large to provide a clear indication of a repulsive force. We need to utilize cavities with dimensions of approximately 100 nm width, 100 nm wall thickness in order to have a clear indication of repulsive forces. The University of Wisconsin Center for NanoTechnology is currently upgrading one of its synchrotron exposure systems in order to provide features of this size.

If we can obtain repulsive as well as attractive vacuum forces by a suitable choice of geometry, we are one step closer to being able to design a variety of novel MEMS devices using vacuum energy that could assist in attaining some of the NASA objectives for space travel.

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